Staffing Policy of Delivery Platforms under the Sharing Economy: A Game-Theoretic Analysis

Completed Research Paper

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Abstract

For companies providing grocery and food delivery, some choose to become two-sided platforms and let part-time people deliver for consumers. While some platforms also hire full-time deliverers, some rely solely on part-time deliverers. It is crucial to explain the difference and identify factors to be considered in making the staffing decision. We consider a delivery platform who may hire full-time deliverers or subsidizing part-time ones to induce their participation. While solving the staffing problem, it also solves the pricing problem with respect to consumers by taking cross-side network externality into consideration. We find that hiring both part-time and full-time employees is suboptimal when the wage for full-time deliverers is too high. Interestingly, hiring only full-time employees is never optimal no matter how low the wage is. Our analysis helps us justify the popularity of delivery platforms and may guide platform deliverers to determine their staffing policies.

Keywords: sharing economy, network externality, grocery and food delivery, staffing policy, game theory.

Introduction

Owing to the great success of Uber, AirBnb, and App Stores, those rapid-growing international giants who put the concept of sharing economy and multi-sided platforms into practice, there are more companies dedicated to developing new business models. Food and grocery delivery is one emerging industry with successful two-sided platforms. In the United States, Instacart is very famous in having contracted shoppers and deliverers to do grocery delivery for customers. In Taiwan, UberEATS, HonestBee, and FoodPanda all adopt the same idea to do food delivery.

All these “platform delivery” companies face similar challenges. First, the balance between food delivery service of demand and the number of contractors (people who take the job when they are available, just like Uber’s driver) is always important. When demand is much higher than supply, customer will have to endure a long waiting time, which causes a rapid decline in user experience. On the other hand, when supply is higher than demand, there will be some contractors having no work to do, and paying them high compensation seems to be unnecessary. The next challenge they may face is that contractors are more difficult to manage with. In fact, different platforms have different staffing policies. While Instacart and HonestBee also hire full-time deliverers, FoodPanda and UberEATS hire part-time deliverers only. How to determine the staffing policy and manage supply is crucial for all platform delivery companies.
In this study, we consider a delivery platform’s staffing strategy in matching demand and supply. While in theory there can be all kinds of mechanisms, in practice three kinds of strategies are common. If a company adopts the all-part-time-contractor strategy (strategy P), the platform will not have any “formal employee”. On the opposite, the platform may only recruit full-time employees. This is the all-full-time-employee strategy (strategy F). Finally, the mixed strategy (strategy M), under which the platform have part-time contractors and full-time employees at the same time. It is worthwhile to investigate which strategy may generate the highest profit for the platform. We hope our study may help explain the rationale behind the selection of strategies adopted by these platforms in practice and provide practical suggestions.

We construct a game-theoretic model featuring network externality and sharing economy to address our research questions. There are three types of players in the market, a firm providing platform delivery service, a group of potential consumers, and a group of potential deliverers. The major purpose of our work is to study the profitability of the three staffing strategies and figure out factors that affect the firm’s choice.

**Literature review**

Uber, a transportation network company, take the concept “sharing economy” into practice and introduce it to the world. Their great success makes many company want to copy their business model. Some people attribute the success of Uber to the problem that how to make good use of idle resources spreading in the market. For instance, Santi et al. (2014) claim that the cumulative trip length could be reduced by roughly 40 percent when using ride sharing like Uber related to traditional taxis. Andersson et al. (2013) investigate ways ride sharing could improve the use of idle resources, and classify the business model of sharing economy into three kinds according to the properties of trade matching. In Felländer et al. (2015), their definition of sharing economy focus on the peer-to-peer exchange of tangible assets and intangible assets which involve information exchange through the Internet or mobile phones. Zervas et al. (2015) analyze the competition relationship between Airbnb and hotel chains. When it comes to the use of idle resources, utilization of resources is one of the most important issues. Both Teresa and Christy (2015) and Rougès and Montreuil (2004) study the paradigm change of crowdsourcing/crowdsourcing delivery, a delivery solution which outsource the delivery business to anyone who is willing to fulfill it. Gurvich et al. (2015) build up a newsvendor model to investigate the benefits of a firm using self-scheduling, which allow its workers/agents decide when and whether to work. Eventually, they arrive at the result below: Self-scheduling can impose excess costs on a firm, and then lower the service level.

However, making good use of idle resources spreading in the market is not the only thing we want to discuss in the sharing economy. The other main topic about those “two-sided market” is network effect, which claims that two agents on different sides of the platform affect each other. According to Katz and Shapiro (1985), the pioneers who study network externality can be classified with three different sources. The main purpose of Jing (2007) is to delve into how network externality impacts on the product line design. In consideration of network externality, Fudenberg and Tirole (2000) study the competition between a monopolistic incumbent and a potential entrant in a two-sided market, and develop a model representing the incumbent’s pricing strategy to deter the threat of entrant. By 2006, there are two trends of literature discussing the pricing strategy with network externality, which are pure membership and pure usage charges. Rochet and Tirole (2006) develop a mixed model combined with these two types of charging methods. Kung and Zhong (2017) conduct a game-theoretic model for platform delivery by addressing both cross-side and same-side network externality. Their finding is that adopting membership pricing increases platform deliverers’ profit in many cases. We follow this stream of literature to apply game-theoretic modeling in the study of two-sided platforms. What makes our study unique is the two options of staffing, where most past studies assume that suppliers or service providers come from a single source.
Problem description and formulation

To avoid confusing, we represent contractor as “shipper” in this section. We first model a platform delivery company’s optimal problem and profit functions of consumers and shippers to depict their behaviors when the platform is a monopolistic company in a market.

Regarding a market exists two groups of agent \(i \in \{C, S\}\), consumers \((i = C)\) and shippers \((i = S)\), and a monopolistic platform (P) who provides platform delivery service to a market and matches consumers and shippers. After matching a transaction successfully, a shipper is in duty bound to deliver groceries to a consumer. In every match, the consumer would pay a transaction price to the platform, and a payment would occur between the shipper and the platform. To simplify, we consider that the platform charges a per matching fee \(p\), and pays the shipper a per transaction fee \(r\).

Because shippers are independent contractors and are not forced to work for the platform, the number of shippers cannot be controlled by the platform. Therefore, the platform may also hire full-time employees directly to ensure the supply of delivery service. Let \(n_F\) be the number of full-time employees and \(W\) be the per-employee wage in a unit of time. Consumers are heterogeneous on their type \(\theta\), the willingness to pay for services. We assumed that \(\theta\) is uniformly distributed in \([0, 1]\). Let \(N > 0\) be the number of orders that a consumer will order in one unit of time, a type \(\theta\) consumer’s utility is thus

\[
\mu_C = N(\theta - p). \tag{1}
\]

In our basic model, we will assume that \(N\) is a constant that is not affected by the prices. While this may be true for services like grocery delivery, this may be inappropriate for services like transportation. To complete a transaction, the shipper incurs a per transaction cost \(\eta\) from the platform, where \(\eta\) is assumed to distribute uniformly within 0 and 1. Therefore, his net earning for completing one transaction is \(-\eta + r\). If there are \(n_C\) customers being members of the platform, there will be in total \(Nn_C\) orders in a membership period. Given that there are \(n_F\) shippers and \(n_P\) hired employee in the market, each shipper in expectation will get \(\frac{Nn_C}{n_F + n_P}\) orders. Therefore, a type-\(\eta\) shipper’s utility in a membership period is

\[
\mu_S = \left(\frac{Nn_C}{n_F + n_P}\right)(-\eta + r). \tag{2}
\]

It is assumed that a consumer or shipper will join the platform if \(\mu_C \geq 0\) or \(\mu_S \geq 0\), respectively. According to our setting, there exists a critical value \(\theta^*\) which divides consumers into two groups: A consumer would join the platform if and only if \(\theta > \theta^*\). Similarly, there exists a critical value \(\eta^*\) such that a shipper would join the platform if and only if \(\eta < \eta^*\). In our notation, this means

\[
n_C = 1 - \theta^* \quad \text{and} \quad n_P = \eta^* \tag{3}
\]

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<tr>
<th>(\theta)</th>
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<td>Not join</td>
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Figure 1: When will the consumers and shoppers join the platform.
In order to differentiate the efficiency of the full-time employee and the part-time contractors, we define $Q_F$ as the number of works that a part-time contractor can finish during a unit of time. Then we define $Q_P$ as the number of works that a full-time employee can finish during a unit of time.

With $Q_P$ and $Q_F$, we can now formulate the platform’s maximum profit

$$\pi = \min\{N(1 - \theta^*), Q_Fn_F\}p + \min\{N(1 - \theta^*) - Q_Fn_F, Q_Pn_P\}(p - r) - Wn_F$$  \hspace{1cm} (4)

The first term of (4) is the revenue earned by full-time employees. As $(1 - \theta^*)$ is the number of consumers willing to use the delivery platform, $N(1 - \theta^*)$ is the total number of orders that consumers may place. However, if there are not enough full-time employees, not all the orders may be fulfilled. In particular, given the number of full-time employee $n_F$, the number of orders that they may fulfilled is $Q_Fn_F$. The total delivery that the platform may complete is thus $\min\{N(1 - \theta^*), Q_Fn_F\}$, the minimum of the demand quantity and supply quantity. Each of these deliveries then gives the platform $p$ as the per-delivery revenue.

The second term of (4) is the revenue earned by part-time contractors. In our basic assumption, if there are more orders that full-time employees can fulfill, the platform may allocate remain orders to part-time contractors. In particular, given the number of part-time contractors $n_P$, the number of orders that they may fulfilled is $Q_Pn_P$. The total delivery that the platform may complete is thus $\min\{N(1 - \theta^*) - Q_Fn_F, Q_Pn_P\}$, the minimum of the remain demand quantity and supply quantity. Each of these deliveries then gives the platform $p - r$ as the per-delivery revenue.

The Last term of (4) is the fixed salary of all the full-time employees, where $W$ represents the salary for each full-time employee in a unit of time.

If a platform wants to maximize its profit, it should solve the above optimization problem with three decision variables $(n_F, p, r)$. As we have mentioned before, different companies in practice adopt different strategies. In this study, we investigate the profitability of three pricing strategies commonly adopted in practice. By adopting strategy P (all-part-time-contractor strategy), the platform does not hired any employee, i.e., $n_F = 0$. On the other hand, under strategy F (all-full-time-employee strategy), the platform does not have any shipper, i.e., $n_P = 0$ and $r = 0$. The third strategy, strategy M (the mixed strategy), lets the platform contracts with the shipper and hires some employee in the same time. We are interested in understanding the profitability of these three strategies.

The sequence of events is as follows. First, the platform decides the per transaction fee $p$, the per matching subsidy $r$, and the hired employee $n_F$. Second, potential consumers and shippers observe the prices of using the service and decide whether to join the platform or not independently. In the end of this stage, the sizes of the two groups will be realized, and the platform can calculate its optimal profit in equilibrium.

Analysis

In this section, we analyze the maximization profit function under three different pricing strategies. First, we present platform’s optimal solution for each strategy, then we compare the best strategy a platform can adapt by comparing their optimal profit under different scenarios.

Optimal profit

We start from the analysis of all-part-time-contractor strategy. First, we derive the utility function of customer and contractor. Given $p, r, n_F = 0$, (1), (2), (3) together imply that

$$\theta^* - p = 0 \text{ and } \frac{N(1 - \theta^*)(r - \eta^*)}{\eta^*} = 0,$$  \hspace{1cm} (5)
where the type $\theta^*$ customer’s and type $\eta^*$ contractor’s utilities to be 0, respectively. Then, we get a solution of $\theta^*$ and $\eta^*$.

$$\eta^* = r \text{ and } \theta^* = p$$  \hspace{1cm} (6)

Substituting $\theta^*$ and $\eta^*$ into (3) and (4), we have the platform’s profit function

$$\pi^P = \max \min \{ N(1-p), Q_pr \} (p-r)$$  \hspace{1cm} (7)

where the platform chooses the per-delivery price $p$ and per-delivery subsidy $r$ to maximize its profit, which is the delivery quantity times the per-delivery margin $p-r$. Note that the delivery quantity is the minimum of the delivery demand $N(1-p)$ and delivery supply $Q_pr$.

The optimal solution can be found as follows.

**Lemma 1.** The optimal solution of all-part-time-contractor strategy is

$$p^P = \frac{(Q_pr+2N)}{2(Q_pr+N)} \text{ and } r^P = \frac{N}{2(Q_pr+N)} \text{ and } \pi^P = \frac{NQ_pr}{4(Q_pr+N)}.$$  \hspace{1cm} (8)

It may be observed that the per-delivery subsidy $r^P$ decreases in the part-time capacity $Q_pr$. When each part-time contractor may serve more consumers, the platform needs fewer part-time contractors. Therefore, it pays less to subsidize these contractors.

Next we consider the all-full-time-employee strategy. Same as the derivation process of the all-part-time-contractor strategy, we can show that the equilibrium number of consumers is still $1 - \theta^* = 1 - p$. However, because the platform now hires no part-time contractor, it will set $r = 0$ and therefore $\eta^* = r = 0$. Substituting $\theta^*$ and $\eta^*$ into (3) and (4), we have the platform’s profit function

$$\pi^F = \max \min \{ N(1-p), Q Fn_F \} p - Wn_F$$  \hspace{1cm} (9)

where the platform maximize its profit by choosing the best per-delivery price $p$ and proper number of full-time employee $n_F$. The profit can be calculated by the delivery quantity times the per-delivery revenue $p$ and minus the total salary for full-time employee $Wn_F$. Note that the delivery quantity is the minimum of the delivery demand $N(1-p)$ and delivery supply $Q Fn_F$.

The optimal solution can be found as follows.

**Lemma 2.** The optimal solution of All-full-time-employee strategy is

$$p^F = \frac{(Q_F+W)}{2Q_F} \text{ and } n_F^E = \frac{N(Q_F-W)}{2Q_F^2} \text{ and } \pi^F = \frac{N((Q_F-W))^2}{4Q_F^2}.$$  \hspace{1cm} (10)

It may be observed that the per-delivery price $p^F$ increases in the salary $W$. When salary becomes higher, the platform needs to add price in order to cover the fixed cost. In contrast, the number of hired
full-time employee $n_F^M$ decrease in the salary $W$, because the platform cannot afford so much employees with expensive salary. We also observe that $p^F$ does not change in the behavior of consumers’ buying frequency $N$, while $n_F^M$ increase in $N$. It may be explained that the behavior of consumers’ buying frequency does not affect per-delivery profit but the total profit. Therefore, the platform can hire more employee to work for them and need not to add any price for delivering. When it comes to the efficiency of full-time employee $Q_F$, we observe that $p^F$ in $Q_F$. It may be explained that the platform can afford more transaction in a period of time, therefore lower the price and attract more consumer may earn more money. However, we observe that $n_F^M$ increase in $Q_F$ at first and then decrease. It may be explained that when the employee’s efficiency is greater, hiring more employee seems to be a great deal to achieve the platform’s full trading capacity. After the threshold of that, the platform just do not need so many employee to work, therefore reduce the number of hired employee.

Finally, we consider the mixed strategy. Same as the deriving process of All-part-time-contractor Strategy, we can show that the equilibrium number of consumers is still $1 - \theta^* = 1 - p$ and number of contractors is still $\eta^* = r$. Substituting $\theta^*$ and $\eta^*$ into (3) and (4), we have the platform’s profit function

$$\pi^M = \max\{\min\{N(1 - p), Q_F n_F\} p + \min\{(N(1 - p) - Q_F n_F)^+, Q_F r\} (p - r) - W n_F\}$$ (11)

where the platform chooses the per-delivery price $p$, per-delivery subsidy $r$ and number of full-time employee $n_F$ to maximize its profit, which is the delivery quantity times the per-delivery revenue $p$ or $p - r$. Then, minus the total salary for full-time employee $W n_F$. Note that the delivery quantity of full-time employee is the minimum of the delivery demand $N(1 - p)$ and delivery supply $Q_F n_F$. If there is remain demand, the delivery quantity of part-time contractors will be the minimum of the remain delivery demand $N(1 - p) - Q_F n_F$ and delivery supply $Q_F r$. Otherwise, it will be zero.

The optimal solution can be found as follows.

**Lemma 3.** The optimal solution of mixed strategy is

$$p^M = \frac{(Q_F + W)}{2Q_F}, \quad n_F^M = \frac{(N(Q_F - W) - W Q_F)}{2Q_F}, \quad r^M = \frac{W}{2Q_F} \quad \text{and} \quad \pi^M = \frac{(N((Q_F - W)^2 + W^2 Q_P))}{4Q_F^2}.$$ (12)

It may be observed that the per-delivery price $p^M$ is absolutely same as $p^F$, therefore it increases in the salary $W$, does not change in the behavior of consumers’ buying frequency $N$ and decrease in the efficiency of full-time employee $Q_F$ for the same reason. When it comes to the number of hired full-time employee $n_F^M$, we observe that it decrease in the salary $W$, increase in $N$ and for the same reason as $n_F^M$. One different is that $n_F^M$ decrease in the efficiency of employees and contractors $Q_F$ and $Q_P$. It may be explained that the equilibrium of the mixed strategy should stand on the basis that there are some contractors cheaper then employee. Therefore, when the efficiency of full-time employee $Q_F$ becomes higher, all the platform that need to do is to hire less employee in order to reduce the cost.

**Comparison**

Up until this point, we have the profit-maximizing prices of three possible pricing strategies. We would like to do some comparison on these strategies to see which one is the platform’s best pricing strategy. Furthermore, we hope our findings could explain the revenue model of platforms in sharing economy, to some extent.

We first observe the variable of mixed strategy that $n_F^M = \frac{N(Q_F - W) - W Q_F}{2Q_F}$ must greater than 0 or the mixed strategy will turn into strategy P for $n_F^M = 0$. Therefore, we know the threshold of employee’s wages to keep mixed strategy work is that
\[ W < W_A = \frac{Q_FN}{N+Q_p} \]  \hspace{1cm} (13)

We examine strategy F by the same rule. We know the threshold of employee’s wages to keep strategy F work, i.e., \( n^F_F > 0 \), is that

\[ W < W_B = Q_F \]  \hspace{1cm} (14)

Then, we can compare three pricing strategy under different scenario of employee’s wages. The results are shown in Proposition 1.

**Proposition 1.** Mixed-strategy is the best strategy if and only if \( W < W_A = \frac{Q_FN}{N+Q_p} \). Strategy P is better than strategy F if and only if \( W \geq W_B = Q_F \). Strategy F is better than strategy P if and only if \( W < \bar{w} = \left(1 - \frac{Q_p}{Q_F+N}\right)Q_F \). It is always true that \( 0 < \bar{w} < W_A < W_B \).

Proposition 1 demonstrates some interesting findings. One straightforward finding is that strategy P is the best if the full-time wage \( W \) is high enough. When it is very expensive to hire full-time employees, the best strategy is not to hire any of them. Strategy P is then optimal. Such a finding may let us intuitively conjecture that when \( W \) goes down, the optimal strategy should change from strategy P to strategy M, and eventually become strategy F. After all, if the full-time wage is low enough, hiring full-time employees should be more efficient. Nevertheless, Proposition 1 shows us our intuition may not always work: When the wage is low enough, mixed strategy is the best strategy. To understand this, note that no matter how low the wage \( W \) is, there are always some contractors (those with \( \eta \) that are close to 0) willing to work for an even lower compensation. Hiring no part-time employee (i.e., adopting strategy F) is never optimal. Finally, Proposition 1 also shows that there exist \( \bar{w} \) that help us decide it is strategy F or strategy P that should be adapted when the platform does not consider mixed strategy for some practical reason.

We provide visualization in Figures 2 and 3. Figure 2 shows us intuitively strategy P takes more advantage when the part-time contractors working efficiency is higher (i.e., \( Q_p \) is high). On the other hand, Figure 3 shows us that strategy P is not suitable for high consumption frequency (i.e., \( N \) is high).

![Figure 2: Optimal Strategy given Salary and Contractor’s Efficiency (\( N = 1, Q_F = 1 \))](image-url)
In practice, we indeed observe the fact that strategy P is more suitable than strategy F when $Q_P$ is high. In Taiwan, FoodPanda began their services without knowing how to manage part-time employees (i.e., $Q_P$ is low). It therefore provided attractive full-time salary which even attracted media coverage few years ago. Their high standard for choosing delivery staffs and strict employee training were well known in our society. After years of developing the training program, FoodPanda found a way to quickly train new part-time employees to have good capability (i.e., $Q_P$ is high). It is thus natural that FoodPanda is now willing to contract with only part-time contractors. Figure 2 also suggests this strategic move.

We also observe that mixed strategy is more suitable than strategy P when $N$ is higher. HonestBee, a startup which had comparative low popularity rate with other food delivery platform when they entered Taiwan, adapted strategy P to be their staffing policy at the beginning. However, after it holds stable customer base in food delivery, HonestBee decided to change their staffing policy to mixed strategy instead of full of part-time contractors. Currently, only deliveries that full-time employees unable to fulfill will be assigned to part-time contractors. This phenomenon is demonstrated in Figure 3, where a more stable customer base represents a higher frequency of consumption $N$.

**Impact of Demand Volatility**

In this section, we consider a case where demand levels at different periods may be different. This may reflect the fact that the demand rates for delivery may vary within a day. More precisely, we assume that within a day sometimes the demand rate is $N_H$ and sometimes it is $N_L$. The proportions of time with these two types of demand rates are $\alpha$ and $1 - \alpha$, respectively. The platform may charge consumers different prices and give part-time contractors different subsidy for the two periods. However, once a full-time employee is hired, she/he will work in both periods with a single wage. We evaluate the two pure pricing strategies to deliver more insights. First, we present platform’s optimal solution for each strategy, then we compare the best strategy a platform can adapt by comparing their optimal profit under different scenarios.

We start from the analysis of all-part-time-contractor strategy. First, we modify equation (7) and (9) with the floating price $p_H$ and $p_L$, the floating transaction fee $r_H$ and $r_L$ and different number of orders $N_H$ and $N_L$ during different time in the period of a day. $p_H$, $r_H$ and $N_H$ represent those floating price, transaction fee and number of orders during the period with higher demand. We can also estimate how long does the high demand period stays in a day by using parameter $\alpha$. Thus, we have our new equation.

Figure 3: Optimal Strategy given Salary and Consumption Frequency ($Q_P = 0.8, Q_F = 1$)
\[
\Pi^p = \max \alpha N_H (1 - p_H)(p_H - r_H) + (1 - \alpha)N_L (1 - p_L)(p_L - r_L) \\
\text{s.t. } N_H (1 - p_H) \leq Q_p r_H \\
N_L (1 - p_L) \leq Q_p r_L
\]
\[
\Pi^f = \max \alpha N_H (1 - p_H)p_H + (1 - \alpha)N_L (1 - p_L)p_L - W n_F \\
\text{s.t. } N_H (1 - p_H) \leq Q_f r_H \\
N_L (1 - p_L) \leq Q_f r_L
\]

It may be noticed that the above equation will be calculated given \( \alpha = 0.5 \), because we want to simplify its mathematical form. The optimal solution can be found as follows.

**Lemma 4.** The optimal solution of all-part-time-contractor strategy and all-full-time-employee strategy with uncertain demand is

\[
p_H^p = \frac{(Q_p + 2N_H)}{2(Q_p + N_H)}, p_L^p = \frac{(Q_p + 2N_L)}{2(Q_p + N_L)}, r_H^p = \frac{N_H}{2(Q_p + N_H)}, r_L^p = \frac{N_L}{2(Q_p + N_L)} \text{ and } \Pi^p = \frac{N_H Q_p}{4(Q_p + N_H)} + \frac{N_L Q_p}{4(Q_p + N_L)}.
\]

\[
p_H^f = \frac{(Q_F + W)}{2Q_F}, p_L^f = 0.5, n_F^p = \frac{N_H (Q_F - W)}{2Q_F} \text{ and } \Pi^f = \frac{N_H (Q_F - W)^2}{4Q_F^2}.
\]

We want to compare this extension result with the previous version. Therefore, we set up a new indicator \( \pi^D = \Pi^p - \Pi^f \) to estimate which strategy dominate the other one. We observe the new indicator in Proposition 2.

**Proposition 2.** Let \( \Pi^D (N_L) = \Pi^p - \Pi^f \) and normalize \( N_H \) to 1 and \( N_L \in [0,1] \). \( \Pi^D (N_L) \) is decreasing in \( N_L \), which implies that \( \Pi^D (N_L^*) = 0 \) has at most one root in \([0,1]\).

Let \( N_L^* \) be the unique solution of \( \Pi^D (N_L) = 0 \). If \( N_L^* > 1 \), strategy P dominates strategy F; if \( N_L^* < 0 \), strategy F dominates strategy P. Finally, if \( 0 < N_L^* < 1 \), the relative performance of the two strategies depends on the value of \( N_L \). In particular, strategy P is more profitable if \( N_L < N_L^* \). In other words, when the two demand levels differ a lot, the flexibility of part-time contractors is quite helpful. On the contrary, strategy F generates more profit if \( N_L < N_L^* \). When the low-demand period is similar enough to the high-demand one, full-time employees may benefit the platform more.

It is also interesting to investigate factors that may influence the cut-off \( N_L^* \). Though the complexity of the model disallows us to analytically characterize the impact of these factors, through numerical observations we still generate managerial implications. First, we observe that \( N_L^* \) is increasing in \( Q_p \), the processing speed of contractors (cf. Figure 4). As contractors becomes more skillful, strategy P becomes more valuable. Second, we observed that \( N_L^* \) is decreasing in \( Q_F \), the processing speed of employees (cf. Figure 5). As employees becomes more skillful, strategy F becomes more valuable. Third, we finally observed that \( N_L^* \) is decreasing in \( W \), the salary level of employees (cf. Figure 6). As employees becomes more expensive to the platform, strategy F may not be profitable for the platform.
Figure 4: $N_L^*$ and Salary $W$ ($Q_P = 0.7, Q_F = 1$)

Figure 5: $N_L^*$ and Contractor’s Efficiency $Q_P$ ($W = 0.6, Q_F = 1$)

Figure 6: $N_L^*$ and Employee’s Efficiency $Q_F$ ($W = 0.6, Q_P = 0.7$)
Conclusion

In this paper, we study staffing policy for delivery service on the two-sided platform. We first formulate the issue into a game-theoretic model with network effect. Then, we calculate the maximum profit under different staffing policy and compare them in some specific scenario. We have the conclusion that adapting mixed strategy earn the most profit when the wage of hiring full-time employee is low enough. Mixed strategy also has the advantage when consumers are more willing to trade on the platform. There are also some intuitive conclusion that strategy P is more likely to be adapted when the working efficiency of part-time contractors is better.

The mechanism we design is simple and comfortable to implement. Therefore, we can attempt to making some business strategies while there are still many future research may be considered. For instance, time-window can be add to the problem in order to limit working hours for full-time employee. Meanwhile, demand may be float in different time-window. The problem with those complicated factors is left open for future research.

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Appendix

Proof for Lemma 1. For maximum profit of strategy P $\pi^P$ in (7), we turn the maximum function to a simpler form with a constraint from the knowledge that when $\pi^P$ is maximum, $N(1-p)$ should equal $Q_pr$. It is because that when $N(1-p) > Q_pr$, we can still add $p$ to maximize the objective value until they are equal. In contrast, when $N(1-p) < Q_pr$, we can still reduce $r$ to maximize the objective value until they are equal. Therefore, the problem becomes

$$\pi^P = Max\;N(1-p)(p-r)$$
$$s.t.\;N(1-p) = Q_pr$$

We then plug $\frac{N(1-p)}{Q_p}$ into $r$ and update $\pi^P = \max\;N(1-p)\left(p - \frac{N(1-p)}{Q_p}\right)$. As it is a concave function, we use first-order condition to find $p^P = \frac{(Q_p+2N)}{2(Q_p+N)}$, $r^P = \frac{N}{2(Q_p+N)}$ and $\pi^P = \frac{NQ_p}{4(Q_p+N)}$. □

Proof for Lemma 2. For strategy F, we also turn the objective function in (9) to a simpler form $N(1-p)p - Wn_F$ with a constraint $N(1-p) = Q_fn_F$ by the same reason we have done in strategy P. Then, we plug $\frac{N(1-p)}{Q_F}$ into $n_F$ and update $\pi^F = Max\;N(1-p)p - W\frac{N(1-p)}{Q_F}$. Obviously, it is also a concave function, therefore we use first-order condition to find $p^F = \frac{(Q_F+W)}{2Q_F}$, $n^F_F = \frac{N(Q_F-W)}{2Q_F^2}$ and $\pi^F = \frac{N(Q_F-W)}{4Q_F}$. □

Proof for Lemma 3. For maximum profit of mixed strategy $\pi^M$ in (11), we can turn the maximum function into two subproblem with different scenario.

$$\pi^M_1 = max\;N(1-p)p - Wn_F$$
$$s.t.\;N(1-p) \leq Q_fn_F$$

$$\pi^M_2 = maxQ_fn_Fp + \min\{[N(1-p) - Q_fn_F]^+,Q_pr\}(p-r) - Wn_F$$
\[ s.t. N(1 - p) \geq Q_F n_F \]

Since \( \pi_1^M \) is same as \( \pi^F \), we can find the maximum value as (10). For the other scenario, we simplify \( \pi_2^M \) with a new constraint for the same reason above.

\[ \pi_2^M = \max Q_F n_F p + (N(1 - p) - Q_F n_F) (p - r) - W n_F \]

\[ s.t. (1 - p) \geq Q_F n_F \]

\[ N(1 - p) - Q_F n_F = Q_p r \]

Then, we plug \( \frac{N(1-p)-Q_F n_F}{Q_F} \) into \( r \) and the problem reduces to

\[ \pi_2^M = \max Q_F n_F p + (N(1 - p) - Q_F n_F) \left(p - \frac{N(1-p)-Q_F n_F}{Q_F} \right) - W n_F \]

\[ s.t. (1 - p) \geq Q_F n_F \]

with only \( p \) and \( n_F \) as decision variables. Therefore we use Hessian matrix to prove that \( \pi_2^M \) is a concave function. Then, we use first-order condition to solve \( p_2^M = \frac{(Q_F + W)}{2Q_F} \), \( n_F^M = \frac{(N(Q_F - W) - W n_F)}{2Q_F} \),

\[ r_2^M = \frac{W}{2Q_F} \] and \( \pi_2^M = \frac{(N(Q_F - W))^2 + W^2 Q_F}{2Q_F^2} \). In order to verify the constraint, we plug \( p_2^M \) and \( n_F^M \) into (19)’s constraint and we get \( \frac{N(Q_F - W)}{2Q_F^2} \geq \frac{(N(Q_F - W) - W n_F)}{2Q_F^2} \) which is correct. Finally, we compare \( \pi_1^M \) and \( \pi_2^M \) to find out that \( p^M = \frac{(Q_F + W)}{2Q_F} \), \( n_F = \frac{(N(Q_F - W) - W n_F)}{2Q_F} \), \( r^M = \frac{W}{2Q_F} \) and \( \pi^M = \frac{(N(Q_F - W))^2 + W^2 Q_F}{4Q_F^2} \).

\[ \square \]

**Proof for Proposition 1.** Since we want to find a \( \bar{W} \) that can help us decide which strategy to choose from, we compare \( \pi^F \) with \( \pi^P \) and find that when \( W > \bar{W} \), \( \pi^P \) is always larger than \( \pi^F \). Conversely, when \( < \bar{W} \), \( \pi^P \) is always smaller than \( \pi^F \). It is trivial to show that \( 0 < \bar{W} < W_A < W_B \).

\[ \square \]

**Proof for Lemma 4.** For strategy \( P \), we turn the objective function in (15) to a simpler form with the constraint \( N_H(1 - p_H) = Q_P r_H \) and \( N_L(1 - p_L) = Q_P r_L \) by the same reason we have done in proof of lemma 1. Then, we plug \( \frac{N_H(1-p_H)}{Q_P} \) into \( r_H \) and \( \frac{N_L(1-p_L)}{Q_P} \) into \( r_L \). Next, update \( \Pi^P \) as \( \max N_H(1 - p_H) \left(p_H - \frac{N_H(1-p_H)}{Q_P} \right) + N_L(1 - p_L) \left(p_L - \frac{N_L(1-p_L)}{Q_P} \right) \). Obviously, it is also a concave function, therefore we use first-order condition to find \( p_H^P = \frac{(Q_P + W)}{2(Q_P + N_H)} \), \( p_L^P = \frac{(Q_P + 2N_L)}{2(Q_P + N_L)} \), \( r_H^P = \frac{N_H}{2(Q_P + N_H)} \), \( r_L^P = \frac{N_L Q_P}{2(Q_P + N_H)} + \frac{N_L Q_P}{4(Q_P + N_L)} \). For strategy \( F \), we also turn the objective function in (16) to a simpler form with the binding constraints. Then, we plug \( \frac{N_H(1-p_H)}{Q_F} \) into \( n_F \). Next, update \( \Pi^F \) as \( \max N_H(1 - p_H) \left(p_H - \frac{W}{Q_F} \right) + N_L(1 - p_L)p_L \). Obviously, it is also a concave function, therefore we use first-order condition to find \( p_H^F = \frac{(Q_F + W)}{2Q_F} \), \( p_L^F = 0.5 \), \( n_F^F = \frac{N_H(Q_F - W)}{2Q_F^2} \) and \( \Pi^F = \frac{N_H(Q_F - W)^2}{4Q_F^2} \).

\[ \square \]

**Proof for Proposition 2.** We can prove that \( \Pi^P(N_L) \) is decreasing in \( N_L \) by evaluating its first-order derivative. The trivial derivation is omitted.

\[ \square \]
References


Teresa, L., L. Christy. 2015. “Crowdsourced delivery,”